I. Oblicz:

1. $\int_0^1 \frac{x-1}{x+1}dx;$ $I = 1-2\ln 2;$

2. $\int_0^a \sqrt{a^2-x^2}\,dx;$ $I = \frac{\pi a^2}{4};$

3. $\int_0^2 \frac{e^{2x}}{1+e^x}dx;$ $I = e^2-1+\ln$

4. $\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^3 x}dx;$ $I = 2\sqrt{2}-2;$

5. $\int_0^{\ln 2} \sqrt{e^x-1}\,dx;$ $I = 2-\frac{\pi}{2};$

6. $\int_0^1 x^3\sqrt{1-x^2}\,dx;$ $I = 0;$

7. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1+\sin x}}dx;$ $I = 2\sqrt{2}-2;$

8. $\int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}dx;$ $I = 1.$

II. Oblicz pole obszaru ograniczonego liniami:

1. $y = \ln x,$ $y = 0,$ $x = e;$ $D = 1;$

2. $y = \ln x,$ $y = \ln^2 x;$ $D = 3-e;$

3. $y = x^2,$ $2x-y+3 = 0,$ $D = \frac{32}{3};$

4. $y = x^2-4x+4,$ $y = x,$ $D = \frac{9}{2};$

5. $y^2 = 2x + 1, \qquad x - y - 1 = 0, \qquad D = \frac{16}{3};$

6. $y^2 = 4x + 4, \qquad y = 2 - x, \qquad D = \frac{64}{3};$

7. $y = \frac{x}{2}, \qquad y = 2x, \qquad y = \frac{2}{x}, \qquad D = 4\ln 2;$

8. $y = x^2, \qquad y = \frac{x^2}{2}, \qquad y = 3x, \qquad D = \frac{27}{2};$

9. $y^2 + 8x = 16, \qquad y^2 - 24x = 48; \qquad D = \frac{32}{3}\sqrt{6};$

III

Obliczyć pole powierzchni bryły obrotowej powstałej przez obrót wokół osi OX krzywej:

1. $y = 2\sqrt{x}, x \in [0, 2]$

2. $y = \cos x, x \in [0, \pi]$

3. $y = \sqrt{x}, x \in [0, 4]$

4. $y = \frac{1}{2}x^4, x \in [0, 2]$

5. $x = t - sint,\ y = 1 - \cos t,\ t \in [0, \frac{\pi}{2}]$

6. $x = 3e^{-t},\ y = 2e^{-t},\ t \in [0, 1]$

7. $x = \ln t,\ y = 2\sqrt{t},\ t \in [1, 2]$

Zad 4. Obliczyć długość łuku krzywej:

4.1. $y = \frac{2}{3}x^{\frac{3}{2}}, x \in [0, 2]$

4.2. $y = \ln(1 - x^2), x \in [\frac{1}{4}, \frac{1}{2}]$

4.3. $y - \operatorname{arc\,tg} x + \operatorname{arc\,tg} \frac{1}{x}, x \in [\frac{\pi}{2}, \pi]$

4.4. $y = \frac{1}{4}x^2 - \frac{\ln x}{2}, x \in [1, 3]$

4.5. $\ln(\cos x), x \in [0, \frac{\pi}{4}]$

4.6. $u = \ln \frac{e^x+1}{e^x-1}, x \in [1, 2]$

4.7. $y = 2\ln \frac{4}{4-x^2}, x \in [0, 1]$

4.8. $y = \arcsin e^{-x}, x \in [0, 3]$

Zad 5. Obliczyć długość łuku krzywej zadanej w postaci parametrycznej:

5.1. $x = t - \cos t\ ,\ y = 1 - \sin t,\ t \in [0, \pi]$

5.2. $x = 4\cos 4t\ ,\ y = 4\sin 4t,\ t \in [0, \frac{\pi}{4}]$

5.3. $x = 2e^{-t}\ ,\ y = 3e^{-t},\ t \in [0, \pi]$

5.4. $x = \sin^3 t\ ,\ y = \cos^3 t,\ t \in [0, \frac{\pi}{3}]$

5.5. $x = t^2\ ,\ y = 3t^3,\ t \in [0, 2]$

5.6. $x = t\sin t + \cos t\ ,\ y = \sin t1 - t\cos t,\ t \in [0, \pi]$

5.7. $x = \frac{4}{3}t^3\ ,\ y = \frac{1}{4}t^2,\ t \in [0, 1]$

VII. Zbadać zbieżność całki niewłaściwej (całki zbieżne obliczyć):

1. $\int_0^1 \frac{1}{x}dx,$ $I = \infty;$

2. $\int_1^2 \frac{x}{\sqrt{x-1}}dx,$ $I = \frac{8}{3};$

3. $\int_1^2 \frac{1}{x\ln x}dx,$ $I = \infty;$

4. $\int_0^3 \frac{1}{\sqrt{9-x^2}}dx,$ $I = \frac{\pi}{2};$

5. $\int_{-1}^1 \frac{x+1}{\sqrt[5]{x^3}}dx,$ $I = \frac{10}{7};$

6. $\int_0^2 \frac{1}{\sqrt[3]{(x-1)^2}}dx,$ $I = 6;$

7. $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}dx,$ $I = -\frac{2}{e};$

8. $\int_{-1}^{\infty} \frac{1}{x^2+2x+2}dx,$ $I = \frac{\pi}{2};$

9. $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}dx,$ $I = e-1;$

10. $\int_0^{\infty} e^{-x}dx,$ $I = 1;$

11. $\int_0^{\infty} xe^{-x^2}dx,$ $I = \frac{1}{2};$

12. $\int_0^{\infty} x^2e^{-x^3}dx,$ $I = \frac{1}{3};$

13. $\int_0^{\infty} x^3e^{-x^2}dx,$ $I = \frac{1}{2};$

14. $\int_0^{\infty} e^{-\sqrt{x}}dx,$ $I = 2;$

15. $\int_1^{\infty} \frac{1}{x^2 + x}dx,$ $I = \ln 2;$

16. $\int_1^{\infty} \frac{1}{x(x+1)^2}dx,$ $I = \ln 2 - \frac{1}{2};$

17. $\int_1^{\infty} \frac{\ln x}{x^2}dx,$ $I = 1.$