

Sprowadzić do postaci kanonicznej równania:

$$\text{a) } \frac{\partial^2 z}{\partial x^2} + 4 \cos 2x \frac{\partial^2 z}{\partial x \partial y} - 4 \sin^2 2x \frac{\partial^2 z}{\partial y^2} - 4 \sin 2x \frac{\partial z}{\partial y} = 0,$$

$$\text{b) } y^4 \frac{\partial^2 z}{\partial x^2} + 2xy^2 \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} + y^2 \frac{\partial z}{\partial y} = 0,$$

$$\text{c) } 9y^4 \frac{\partial^2 z}{\partial x^2} - 6y^2 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} - 6 \frac{\partial z}{\partial y} = 0,$$

$$\text{d) } \frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + a \frac{\partial z}{\partial x} + b \frac{\partial z}{\partial y} + cz = 0,$$

$$\text{e) } y \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0, \text{ dla } y < 0.$$

$$\text{f) } x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = 0,$$

$$\text{g) } \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0,$$

$$\text{h) } x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0,$$

$$\text{i) } y^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + 2x^2 \frac{\partial^2 z}{\partial y^2} + y \frac{\partial z}{\partial y} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} - 3 \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + 5 \frac{\partial^2 z}{\partial y^2} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} - 2 \sin x \frac{\partial^2 z}{\partial x \partial y} - \cos^2 x \frac{\partial^2 z}{\partial y^2} - \cos x \frac{\partial z}{\partial y} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} - 3 \frac{\partial^2 z}{\partial y^2} + 2 \frac{\partial z}{\partial x} + 6 \frac{\partial z}{\partial y} = 0,$$

$$\bullet \frac{\partial^2 z}{\partial x^2} + 4 \frac{\partial^2 z}{\partial x \partial y} + 5 \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 2 \frac{\partial z}{\partial y} = 0,$$

- $\frac{\partial^2 z}{\partial x^2} - 2 \cos x \frac{\partial^2 z}{\partial x \partial y} - (3 + \sin^2 x) \frac{\partial^2 z}{\partial y^2} - y \frac{\partial z}{\partial y} = 0,$
- $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} - 3y^2 \frac{\partial^2 z}{\partial y^2} + 2x \frac{\partial z}{\partial x} + 4y \frac{\partial z}{\partial y} + 16x^4 z = 0,$
- $\sin^2 x \frac{\partial^2 z}{\partial x^2} - 2y \sin x \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0.$

Wyznaczyć rozwiązanie ogólne równania:

- a) $\frac{\partial^2 z}{\partial x^2} - 2x \frac{\partial^2 z}{\partial x \partial y} - 8x^2 \frac{\partial^2 z}{\partial y^2} - \frac{6x^2 + 1}{x} \frac{\partial z}{\partial x} - 12x^2 \frac{\partial z}{\partial y} = 0,$
- b) $x^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0,$
- c) $xy^3 \frac{\partial^2 z}{\partial x^2} + x^3 y \frac{\partial^2 z}{\partial y^2} - y^3 \frac{\partial z}{\partial x} - x^3 \frac{\partial z}{\partial y} = 0,$
- d) $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} - 2y \frac{\partial z}{\partial y} = 0,$
- e) $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} - 3 \frac{\partial^2 z}{\partial y^2} = 0,$
- f) $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} - 3y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0,$
- g) $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} = 0,$
- h) $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0,$
- i) $4y^2 \frac{\partial^2 z}{\partial x^2} - e^{2x} \frac{\partial^2 z}{\partial y^2} - 4y^2 \frac{\partial z}{\partial x} + \frac{e^{2x}}{y} \frac{\partial z}{\partial y} = 0,$
- j) $x^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0,$